

Notes on Indefinite Metric Theory of Gupta-Bleuler

we start from commutation relations

$$[a_\mu(k), a_\nu^\dagger(k')] = -\delta_{\mu\nu} \delta(k-k')$$

So for $\mu = 1, 2, 3$, eq. $[a_i, a_i^\dagger] = 1$ (1)

but $[a_4, a_4^\dagger] = -1$ (2)

Hamiltonian takes the form

$$H = \sum_k A_\mu^\dagger(k) A_\mu(k) = \sum_k \{ A_1^\dagger A_1 + A_2^\dagger A_2 + A_3^\dagger A_3 - A_4^\dagger A_4 \}$$

$$= \sum_k (n_1 + n_2 + n_3 - n_4)$$

where $n_i = A_i^\dagger A_i$, etc. and $n_4 = A_4^\dagger A_4$ as well.

In order, that we treat (2) by assuming

a_4 is creation operator, a_4^\dagger destruction.

We now try to keep assumption that a_4^\dagger is creation operator, and consider how states

$$\frac{1}{n!} a_4^\dagger(k_1) a_4^\dagger(k_2) \dots a_4^\dagger(k_n) |\bar{\Psi}_0\rangle$$

consider $|n, i\rangle = a_4^\dagger(k_i) |\bar{\Psi}_0\rangle$

($\frac{1}{n!}$ for $k_1 = k_2 = \dots = k_n$ at any rate)



Then $\langle \psi_1 | \psi_1 \rangle = \langle \psi_0 | a_4 a_4^\dagger | \psi_0 \rangle$ 2
 $= \langle \psi_0 | -1 + a_4^\dagger a_4 | \psi_0 \rangle$
 $= -1$ if $\langle \psi_0 | \psi_0 \rangle = 1$
 and $a_4 | \psi_0 \rangle = 0$

Similarly $\langle \psi_2 | \psi_2 \rangle = \frac{1}{2} \langle \psi_0 | a_4 a_4 a_4^\dagger a_4^\dagger | \psi_0 \rangle$
 $= \frac{1}{2} \langle \psi_0 | a_4 (-1 + a_4^\dagger a_4) a_4^\dagger | \psi_0 \rangle$
 $= \frac{1}{2} \langle \psi_0 | -1(-1 + a_4^\dagger a_4) + a_4 a_4^\dagger (-1 + a_4^\dagger a_4) | \psi_0 \rangle$
 $= \frac{1}{2} \langle \psi_0 | 1 + 1 | \psi_0 \rangle = 1$

In general $\langle \psi_n | \psi_n \rangle = (-1)^n$.

So basis states have negative norm for odd n .

Hence $\psi = \sum a_n | \psi_n \rangle$

$\langle \psi | \psi \rangle = \sum a_n^* a_n \langle \psi_n | \psi_n \rangle$
 $= \sum (-1)^n a_n^* a_n$ which is

not positive definite, but corresponds to an indefinite metric in the system space

for n modes $f_{\mu\nu}$, for oscillator 4

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$$\langle 4|4 \rangle = \sum g_{\mu\nu} a_{\mu}^{\dagger} a_{\nu}, \text{ where}$$

$g_{\mu\nu}$ is metric tensor $\langle \psi_{\mu} | \psi_{\nu} \rangle$ and the

form is positive definite in ordinary Hilbert space of Q. Mechanics.

We now obtain properties of number operator

Eigenvalues of $N_4 = a_4^{\dagger} a_4$

Consider $|\psi_1\rangle = a_4^{\dagger} |\psi_0\rangle$

$$\text{for } N_4 |\psi_1\rangle = a_4^{\dagger} a_4 a_4^{\dagger} |\psi_0\rangle$$

$$= a_4^{\dagger} [-1 + a_4^{\dagger} a_4] |\psi_0\rangle$$

$$= -1 \times a_4^{\dagger} |\psi_0\rangle$$

$$\text{Similarly } N_4 |\psi_n\rangle = a_4^{\dagger} a_4 \cdot \underbrace{a_4^{\dagger} a_4^{\dagger} \dots a_4^{\dagger}}_{n \text{ times}} |\psi_0\rangle$$

$$= \left(a_4^{\dagger} (-1) \underbrace{a_4^{\dagger} \dots a_4^{\dagger}}_{n-1 \text{ times}} + \underbrace{a_4^{\dagger} a_4^{\dagger} \dots a_4^{\dagger}}_{n \text{ times}} a_4 a_4^{\dagger} \right) |\psi_n\rangle$$

$$\rightarrow -n |\psi_n\rangle$$

Hence, N_4 has eigenvalues $-N_4$

$$\therefore \langle 1 | 1 \rangle = \sum_n \langle 1 | n \rangle \langle n | 1 \rangle = \sum_n \langle 1 | n \rangle \langle n | 1 \rangle = 1$$

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and these exponents of H and \bar{H} are positive definite.

Also a_4 & d_4^T can be repeated in
lines of the (4x) by notes on them
→ David, Reich, p. 100

$$A_4 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -\sqrt{2} & 0 \\ 0 & 0 & 0 & -\sqrt{3} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_4^* = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

{No op is not an even number} \rightarrow operator in effect -

Stress, σ is not equal $(\sigma_3 - \sigma_4) / 4 = 0$

$$|\psi\rangle = |\psi_0\rangle + \sum_{n=1}^{\infty} |\psi_n\rangle$$

conjugate to

$$\frac{\partial A_1(+)}{\partial x}(\psi) = 0$$

where in its formula with.

NP2 is
inverted
craft.

$$|2\rangle = \sum_n \left(\frac{1}{n!} \right)^{\frac{1}{2}} |n\rangle$$

$$\text{Se } n_3 + n_4 = n$$

Since $\langle \psi | \psi \rangle = \langle \psi | \psi \rangle = 1$ for all ψ 's of the form (born in the laboratory L_4) — different members of (L_4) , corresponding to different groups, give the same result, so we can take

Boundary conditions on ~~ψ~~ $a_3 |\psi_0\rangle = 0$ 5
 $a_1 |\psi_0\rangle = 0$

Expectation values of position are found between
 all base vectors ψ_n with negative norms
 for odd n , no ordering way.

On new departures in formalism
 is that we must not assume $\langle \psi_n | \psi_n \rangle = 1$
 is possible for all base vectors, i.e.
 we are not dealing with an orthonormal
 Hilbert space.

We note on end that if
 $\frac{\partial A_{el}^+}{\partial u} |\psi_0\rangle = 0$ then full brackets
 condition is satisfied for expectation values,
 then $\langle \psi_0 | \frac{\partial A_{el}^+}{\partial u} + \frac{\partial A_{el}}{\partial u} | \psi_0 \rangle$
 $= \langle \psi_0 | \frac{\partial A_{el}^+}{\partial u} | \psi_0 \rangle + \langle \frac{\partial A_{el}}{\partial u} \cdot \psi_0 | \psi_0 \rangle$
 as both terms are zero.

Notice that full Lorentz condition $(L^+ + L^-)|\psi_0\rangle = 0$

$$\left. \begin{aligned} \text{requires with } (a_3 - a_4)|\psi_0\rangle &= 0 \\ \text{and } (a_3^+ - a_4^+)|\psi_0\rangle &= 0 \end{aligned} \right\}$$

is combined with the softer requirement
of $a_3 - a_4|\psi_0\rangle = 0$

Note 1 x/s
in } is constant
alt. notes for
Hawards
systems from the state of Pa
⇒

Notes on Gasiorowicz

Elementary Particle Physics

S-matrix introduced in Heisenberg's formalism

Note definition $S_{\alpha\beta} = \langle \psi_{\alpha}^{\text{out}} | \psi_{\beta}^{\text{in}} \rangle$

this corresponds to $|\psi_{\beta}^{\text{in}}\rangle = \sum_{\alpha} S_{\alpha\beta} |\psi_{\alpha}^{\text{out}}\rangle$

$|\psi^{\text{out}}\rangle$ is eigenstate of $H_0(+\infty)$

$|\psi^{\text{in}}\rangle$ is eigenstate of $H_0(-\infty)$

and are associated
eigenstates of H (total
energy) cf. Schwinger.

Define operator S by $|\psi_{\alpha}^{\text{in}}\rangle = S |\psi_{\alpha}^{\text{out}}\rangle$

then $S_{\alpha\beta} = \langle \psi_{\alpha}^{\text{in}} | S | \psi_{\beta}^{\text{in}} \rangle$

$= \langle \psi_{\alpha}^{\text{out}} | S | \psi_{\beta}^{\text{out}} \rangle$

$= \langle \phi_{\alpha} | S | \phi_{\beta} \rangle$

where ϕ_{α} is eigenstate of H_0 ^{eigenstates}
w. of $H_0(0)$

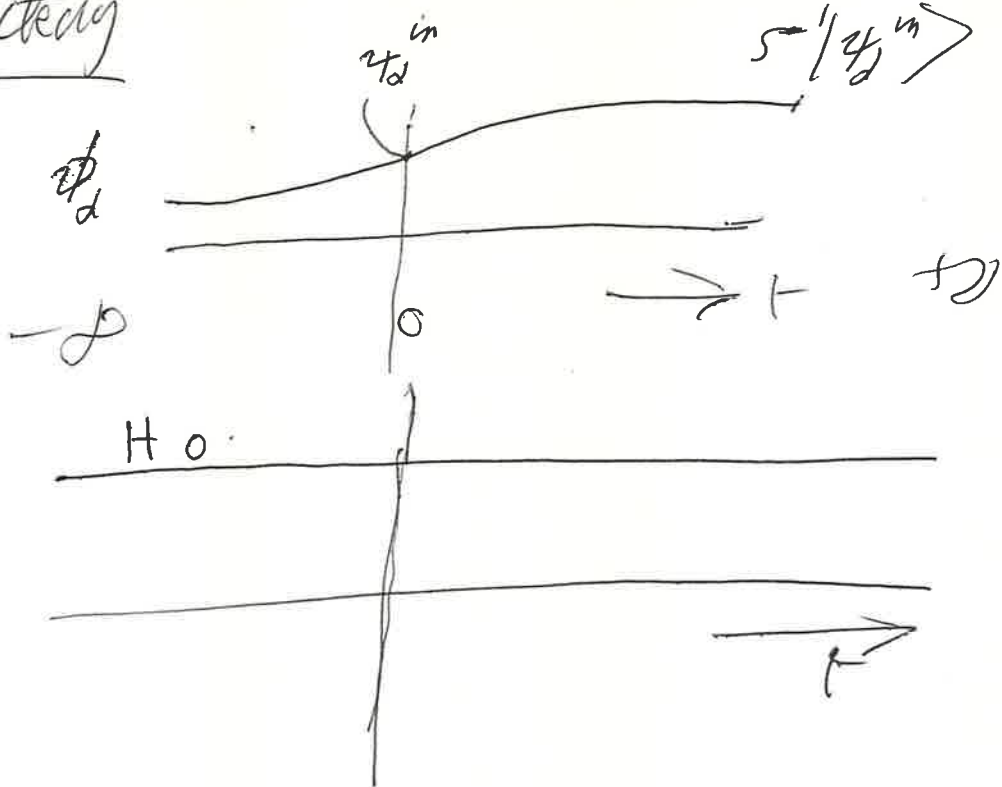
$S = U(\infty, -\infty)$ is unit S-matrix operator.

We can then prove that $S = S^{\dagger}$, from relations

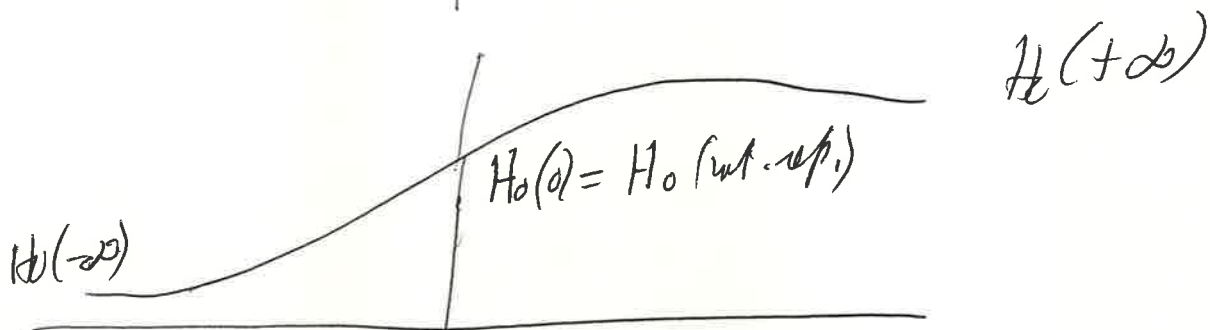
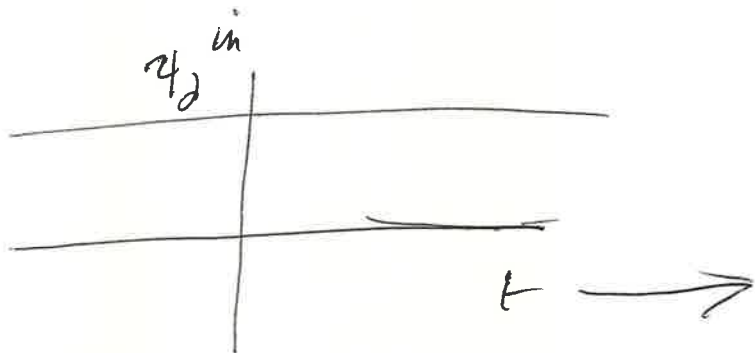
between $\psi_{\alpha}^{\text{in}}$ & ϕ_{α} e.g. $|\psi_{\alpha}^{\text{in}}\rangle = U(0, -\infty) |\phi_{\alpha}\rangle$

Relationship between I.R. & Heaviside, rep. for scattering theory

Int. Rep.



Heaviside rep.



check $H(-\infty)$ ($2\phi_d^{in}$) is opposite of $H(-\infty)$
 just as $1/\phi_d$ is opposite of H_0

Reduction formulae L.S. 2 for scattering from initial $\frac{2}{2}$
 2-particle state to final state λ

$$\begin{aligned} S_{\lambda; pq} &= \langle \psi_{\lambda}^{\text{out}} | \psi_{pq}^{\text{in}} \rangle \\ &= \lim_{t \rightarrow -\infty} \langle \psi_{\lambda}^{\text{out}} | A^{\dagger}(p, t) | \psi_q \rangle \\ &= \lim_{t \rightarrow -\infty} \langle \psi_{\lambda}^{\text{out}} | i \int_{x_0=t} d^3x \phi^*(x) \overleftrightarrow{\partial}_0 f_p(x) | \psi_q \rangle \end{aligned}$$

now write $\int_{x_0=-\infty} d^3x = \int_{x_0=0} d^3x - \int dx \frac{2}{\partial x_0} []$

1st term gives $\langle \psi_{\lambda}^{\text{out}} | \psi_{pq}^{\text{out}} \rangle = S_{\lambda, pq} = (1)_{\lambda, pq}$.

where $(S-1)_{\lambda, pq} = i \int d^4x \langle \psi_{\lambda}^{\text{out}} | J^{\dagger}(x) | \psi_q \rangle f_p(x)$

where $(\square^2 + \mu^2) \phi(x) = J^{\dagger}(x)$ is current source
 of meson field. We can then go from $J^{\dagger}(x)$

$\rightarrow J^{\dagger}(0)$ as before by proper integration

thus $J(x) = e^{i p_{\mu} x_{\mu}} J(0) e^{-i p_{\mu} x_{\mu}}$

$$\begin{aligned} \text{so } \langle \psi_{\lambda}^{\text{out}} | J^{\dagger}(x) | \psi_q \rangle f_p(x) &\rightarrow \langle \psi_{\lambda}^{\text{out}} | J^{\dagger}(0) | \psi_q \rangle \\ &\times \int d^4x e^{i(p_{\lambda} - p_q - p_p) \cdot x} \\ &= \delta^4(p_{\lambda} - p_q - p_p) \langle \psi_{\lambda}^{\text{out}} | J^{\dagger}(0) | \psi_q \rangle \end{aligned}$$

or in q_i notation $\delta^4(p_{\lambda} - p_i) (\psi_{\lambda}^{\text{out}}, J^{\dagger}(0) \psi_q)$

$$\langle \phi | (a^\dagger a) | \phi \rangle = \langle \phi | a^\dagger a | \phi \rangle$$

$$\langle \phi | (a^\dagger a) | \phi \rangle = \langle \phi | a^\dagger a | \phi \rangle$$

$$\langle \phi | a^\dagger a | \phi \rangle = \langle \phi | a^\dagger a | \phi \rangle$$

When we have ψ to ϕ

by normal

By repeated use we can extract "contact" terms field operators
 whilst we are left a normal ordered ketbra $\langle \psi_0 | \dots | \psi_0 \rangle$

In perturbation theory we have to reduce
 expressions of the form $\langle \psi_0 | \hat{A}(x_1) \dots \hat{A}(x_n) | \psi_0 \rangle$

\hat{A} satisfy ordinary canonical commutation relations
 we express \hat{A} in terms of $A^{(0)}$ the solution
 of the equations at $t=0$.

we write $\hat{A}(x) = U(t,0) A^{(0)}(x) U$

$$A^{(0)}(x) = U(t,0) \hat{A}(x) U^{-1}(t,0) \\ = U(t,0) \hat{A}(x) U^{\dagger}(t,0)$$

$$\text{hence } \hat{A}(x) = U^{\dagger}(t,0) A^{(0)}(x) U(t,0) \quad \begin{matrix} U(t_1,0) \\ U^{-1}(t_2,0) \\ \uparrow = U(t_1,0)U(t_2,0) \\ = U(t_1,t_2) \end{matrix}$$

$$\text{then } \langle \psi_0 | \hat{A}(x_1) \dots \hat{A}(x_n) | \psi_0 \rangle = \langle \psi_0 | U^{\dagger}(t_1,0) A^{(0)}(x_1) U(t_1,0) U^{\dagger}(t_2,0) \\ A^{(0)}(x_2) U(t_2,0) \dots \psi_0 \rangle = \langle \psi_0 | U^{\dagger}(t_1,0) A^{(0)}(x_1) U(t_1,t_2) \\ A^{(0)}(x_2) \dots U(t_n,0) | \psi_0 \rangle$$

$$A^{(0)}(x_2) \dots U(t_n,0) | \psi_0 \rangle$$

$$A^{(0)}(x_2) \dots U(t_n,0) | \psi_0 \rangle$$

$$\text{So } \langle \psi_0 | T(\hat{A}(x_1) \dots \hat{A}(x_n)) | \psi_0 \rangle = \lim_{\substack{t \rightarrow \infty \\ t' \rightarrow -\infty}} \frac{\langle \psi_0 | T(U(t,t') A^{(0)}(x_1) \dots A^{(0)}(x_n)) | \psi_0 \rangle}{\langle \psi_0 | U(t,t') | \psi_0 \rangle}$$

Strangeness

we have.

for pions $Q = T_3$

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for baryons

$$Q = \frac{1}{2} N_B + T_3$$

$N_B \leftarrow$ baryon no

The general relation between charge & T_3 is

$$Q = \frac{1}{2} N_B + \frac{1}{2} S + T_3$$

(Gell-Mann 1953)

$S=0$ for pions & nucleons.

$S=-1$ for Λ^0

K mesons produced in association with Λ^0 must

have $S=1$ (e.g. $K^+ \rightarrow \bar{K}^0$ $S=1$
 $K^- \rightarrow \bar{K}^0$ $S=-1$)

(note reaction $n+p \rightarrow \Lambda^0 + \Lambda^0 + \pi^+ + \dots$

is forbidden)

for Σ^0 $S=-1$

Hypercharge Y is defined by

$$Y = \frac{1}{3} (N_B + S)$$

$$\text{so } Q = Y + T_3$$

Ξ^0 $S=-2$

$\frac{1}{2} T$: as parent of
the infinitesimal part.
T : as the whole part. After
pieces of
Molecules
in $\frac{1}{2} T$: makes transitions
remains in the T's as homotopy

Unitary symmetry

starts with isotopic spin formalism

$\frac{1}{2} \tau_i$ yields rotation through angles $\alpha_i = \pi_i \theta$
 θ is angle of rotation
 π_i is isotopic spin vector
 θ is angle of rotation

infinitesimal transformation is

$$1 + \frac{i}{2} \tau_i d\alpha_i$$

\rightarrow 2+2 unitary unimodular transformations

generalization \rightarrow 3+3 unitary unimodular = $(2 \times 9) - (9+1)$
 $=$ 8 - dimensional parameter group

infinitesimal generator is $1 + i \sum_{i=1}^8 d_i F_i$

then F_i satisfy commutation relations $[F_i, F_j] = i f_{ijA} F_A$

$SU(3)$ is a Lie group of rank 2 - two

commuting generators - can be diagonalized simultaneously

- Take T_3 and Y for the generators

Take states which support irreducible rep. - labelled by

eigenvalues of T_3 & Y in 2-dimensional plot

shift operators change the eigenvalues.

- For various values of T_3 & Y we

are with out set of companion states - trace out

hexagonal boundary + interior points



Represent Jan as a Δ - gives 3 representations.

will at reduction of tensor product of

irreducible reps $8 \times 8 = 1 + \overbrace{3+3}^{8+8+10+} + 10 + 27$

Reduction coefficients are Clebsch-Gordan coefficients.

[Note on rotations

$$A_{\mu\nu} \approx \delta_{\mu\nu} + \epsilon_{\mu\nu\lambda} \pi_\lambda \theta \quad \text{small } \theta$$

$$\text{Hence } \chi'_\mu = A_{\mu\nu} \chi_\nu = \chi_\mu + \epsilon_{\mu\nu\lambda} \pi_\lambda \chi_\nu \theta \quad (1)$$

$$= \chi_\mu + \left(\frac{1}{2} + \frac{1}{2} \right) \theta$$

we can write this as

$$A_{\mu\nu} \approx \delta_{\mu\nu} + w_{\mu\nu}$$

$$w_{\mu\nu} = \epsilon_{\mu\nu\lambda} \pi_\lambda \theta$$

for rotation $w_{\mu\nu}$

$$\text{so } w_{12} = -w_{21} = \pi_3 \theta$$

orthogonal

which is what θ states

2 axes.

For rep. of rotation group.

$$M \rightarrow R(\vec{n}, \theta)$$

for infinitesimal element

$$M = 1 + \frac{\partial M}{\partial w_{\mu\nu}} w_{\mu\nu}$$

$$M_{ij} = 1 + S_{ij}^{\mu\nu} w_{\mu\nu}$$

$$= 1 + S_{\mu=1,2,3}^{\mu\nu} \pi_\mu \theta$$

$$= 1 + S_{\mu}^{\mu\nu} \epsilon_{\mu} \quad \epsilon_{\mu} = \pi_{\mu} \theta$$

Invariant coupling in 8-plet way

3

is generalization of $\bar{\psi}_a (\Gamma_i)_{ab} \psi_b \phi_i$

$$\rightarrow \bar{\psi}_m (F_i)_{mn} \psi_n \phi_i = -i f_{imn} \bar{\psi}_m \psi_n \phi_i$$

using representation $(F_i)_{mn} = -i f_{imn}$.

$$\text{But } f_{imn} = \frac{1}{4i} \text{Tr}([\lambda_m, \lambda_n] \lambda_i)$$

$$\text{So coupling takes form} = \frac{1}{4} \text{Tr}([\lambda_m \bar{\psi}_m, \lambda_n \psi_n] \lambda_i \phi_i)$$

$$\bar{B} = \frac{1}{\sqrt{2}} \lambda_m \bar{\psi}_m \quad B = \frac{1}{\sqrt{2}} \lambda_n \psi_n \quad \text{Tr}([\bar{B}, B] M)$$

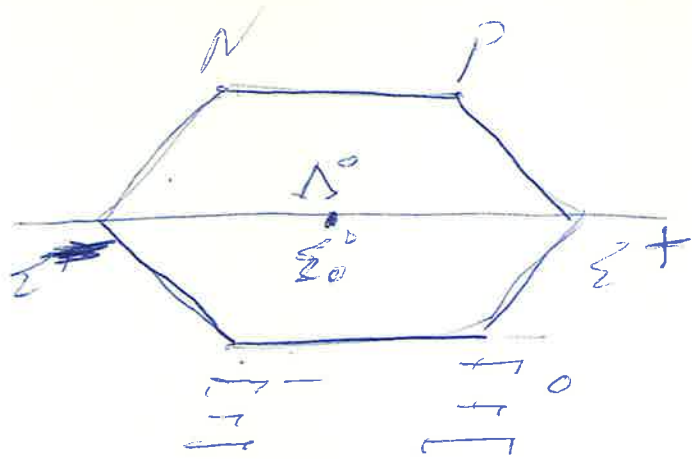
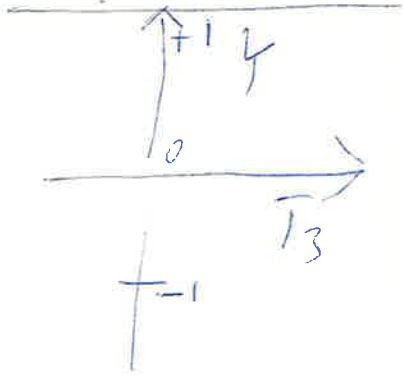
$M = \frac{1}{\sqrt{2}} \lambda_i \phi_i$ 8 components of ψ or ϕ_i

2nd type of invariant coupling is $\text{Tr}(\{\bar{B}, B\} M)$

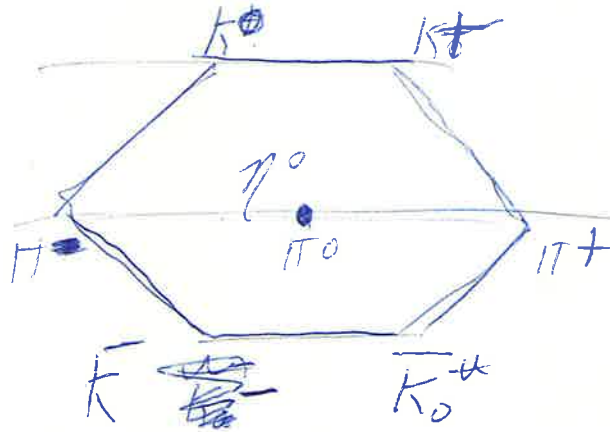
general Yukawa coupling is written as

$$= (1-\alpha) \text{Tr}([\bar{B}, B] M) + 2 \text{Tr}(\{\bar{B}, B\} M)$$

Baryon octet



Meson octet



Two doublets

4

Triplet

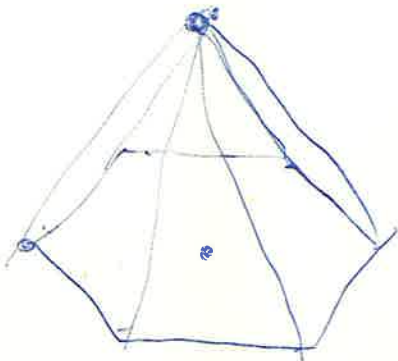
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singlet

1

8

particles in all.



Connection between state & field ^{version} ~~schemes~~
 of higher symmetry schemes

If $U \phi_\alpha U^{-1} = F_{\alpha\beta} \phi_\beta$ for transformation U

then single particle states $\phi_\alpha |\psi_0\rangle$ also
 support the rep. $F_{\alpha\beta}$ — this

~~$U \phi_\alpha U^{-1}$~~ $U \phi_\alpha |\psi_0\rangle = U \phi_\alpha U^{-1} U |\psi_0\rangle$
 $= F_{\alpha\beta} \phi_\beta |\psi_0\rangle$ — ①

so the states $\phi_\alpha |\psi_0\rangle$ support
 the rep.

conversely if $\phi_\alpha |\psi_0\rangle$ support a rep.

① then $U \phi_\alpha U^{-1} = F_{\alpha\beta} \phi_\beta$, assuming
 always that $U |\psi_0\rangle = |\psi_0\rangle$

Note on Infinitesimal Generators of Symmetry Transformations

$$U = 1 + iK \approx e^{iK} \quad \text{if } K \text{ is generator}$$

Then $i[K, \phi]$ gives change in field ϕ
which is invariant under symmetry canonical relations
& field equations remain ($\phi' = \phi + i[K, \phi]$)

We also require Schrödinger Eq to
be unchanged in form hence $[K, H] = 0$

In S. representation, the basis is
eigenstates of H satisfying invariance w.r.t K
as in N.P. theory (original of S.
equation being invariant in Wigner's treatment
e.g.)

If we work in interaction rep. we
require $[K, H'] = 0$, H' is interaction

total energy - this is equivalent
to $[K, S] = 0$ in Dirac

Populal's treatment.

Note that $\{K, H\} = 0$ is not equivalent
for field equations to be equivalent
— this follows from equivalence of V .

But $\{K, H\} = 0$ implies K is
constant in time, and can be
identified with constant of motion from
Noether's theorem, using conservation
properties, which we then do derive
from invariance of field equations
via. form invariance of the Lagrangian
density — this difference often appears
to be overlooked.

Every P.C. is not about invariance in
the S representation. — 2. to Int. ref.

$\{K, H\}$ is equivalent to invariant Torsionless
— stronger equations
p. 12.

proceeding.

$H'(A)$ has a section in $\mathcal{H}(A)$ if and only if $[K, H'] = 0$ when $\mathcal{H}(A)$ is out.

Baryon resonances

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Dalitz plot

3 particle reaction such as
 $K^- + p \rightarrow \Lambda^0 + \pi^+ + \pi^-$ plot $(w_2 + w_3)^2 - (p_2 + p_3)^2$
 against $(w_1 + w_2)^2 - (p_1 + p_2)^2$ e.g. peaks at M^2
 of unstable particle if resonance R is produced.

note $(w_2 + w_3)^2 - (p_2 + p_3)^2 \propto w_1^2$ - energy of 3rd

particle, so Dalitz plot will show events as
 a function of energies of two of the emerging particles.

1st ~~Baryon~~ ^{Baryon} resonance to be discovered
 Resonance occurs in $\pi^+ + p \rightarrow \pi^+ + p$ (also $\pi^- + p$)

a number of which occurs at 1238 MeV
 1518 MeV, 1640 MeV, 1922 MeV & 2204 MeV
 all have $Y=1$ (hypercharge anti) $S=0$

Also strange resonances, $Y=0$ & $Y=-1$ all occur.

Discovery of the Ω^- as 1-spin singlet of negative charge
 and $Y=-2$, $S=-3$ gives $Q = \frac{1}{2} - \frac{3}{2} = -1$

predicted as tenth member of + 10 (0,3) representation
 of $SU(3)$.

P meson resonances

5

ρ meson

observed. $\pi^- + p \rightarrow \pi^+ + \pi^- + n$.

(inelastic process)

(π^+, π^-) peak.

ω meson

observed in $p + \bar{p} \rightarrow \pi^+ + \pi^+ + \pi^0 + \pi^- + \pi^-$

sharp peak for $(\pi^+ \pi^0 \pi^-)$

ϕ meson

$\rightarrow K K$ carrier

decay of ρ meson is allowed
by strong interaction

ρ meson

is ρ meson

in the reaction

$$\pi^- + p \rightarrow \pi^0 + \pi^+ + \pi^- + \pi^+ + p$$

a four particle peak. we found correlation

$\pi^+ \pi^0 \pi^-$ strongly correlated at the

ω meson mass

ρ meson is interpreted as a $(\omega \pi)$ resonance.

Weak Interactions:

form of the β -interaction, non-conservation of parity from experiments of Wu, α β correlation between \vec{p}_e & $\vec{\sigma}$ for decay of polarized nucleus. $\vec{p}_e \cdot \vec{\sigma}$ is a pseudoscalar quantity

If initial state is of even parity, final state

$$\psi_f = f \psi_{\text{odd}} + (1-f) \psi_{\text{even}} \quad \text{say.}$$

$$\langle \vec{p}_e \cdot \vec{\sigma} \rangle_f = \frac{2f(1-f) \langle \psi_{\text{odd}} | \vec{p}_e \cdot \vec{\sigma} | \psi_{\text{even}} \rangle}{f^2 + (1-f)^2}$$

$$\propto \frac{f(1-f)}{f^2 + (1-f)^2} \quad \text{is zero if } f=0 \text{ or } 1$$

i.e. vanishing $\langle \mathcal{O}_{\text{pseudoscalar}} \rangle$ means vector & even 2

odd parity states is required

Note that $\langle \psi_{\text{even}} | \mathcal{O}_{\text{pseudoscalar}} | \psi_{\text{odd}} \rangle$ is non-vanishing

But $\langle \psi_{\text{even}} | \mathcal{O}_{\text{pseudoscalar}} | \psi_{\text{even}} \rangle$ e.g. vanishes

$$= \langle \psi_{\text{even}} | \bar{P}^\dagger P \mathcal{O} P^{-1} P | \psi_{\text{even}} \rangle$$

$$= - \langle \psi_{\text{even}} | \mathcal{O} | \psi_{\text{even}} \rangle$$

$$\text{where } \langle \psi_{\text{even}} | \mathcal{O} | \psi_{\text{even}} \rangle = 0$$

$$\text{Since } P^2 = 1$$

$$P^{-1} = P$$

and $P = P^\dagger$
unitary operator

the result follows

Note that P_\pm is now a field momentum operator as

$$\text{So } P Q(x) P^{-1} = -Q(-x) \text{ but in fact to be interpreted over all } x \text{ as } \int d^3x Q(-x) = \int d^3x Q(x)$$

and changing $x \rightarrow -x'$

Universal Fermi Interaction replaces $\pi \rightarrow e + \bar{\nu}$ e.g.

$$\pi \rightarrow \bar{p} + n \rightarrow e + \bar{\nu}$$

$$\text{Proposed form of } H_w = \frac{G}{\sqrt{2}} \int d^3x \bar{J}_\alpha(x) J^{\alpha+}(x)$$

$$\text{with } J_\alpha(x) = \bar{\psi}_0 \gamma_\alpha (1 - \gamma_5) \psi_\nu + \bar{\psi}_\mu \gamma_\alpha (1 - \gamma_5) \psi_\nu + \bar{\psi}_n \gamma_\alpha (1 - \gamma_5) \psi_p + \dots$$

Could be interpreted as due to a heavy intermediate boson
 $m_w \gg 2 \text{ BeV}$.

$$\text{We can also write } J^\alpha(x) = J_V^\alpha(x) + J_A^\alpha(x)$$

$$J_V^\alpha \text{ is leptonic current } \bar{\psi}_e \gamma^\alpha (1 - \gamma_5) \psi_\nu + \bar{\psi}_\mu \gamma^\alpha (1 - \gamma_5) \psi_\nu$$

$$J_{\text{strong}}^\alpha = J_{V,0}^\alpha + J_{A,0}^\alpha = \bar{\psi}_n(x) \gamma^\alpha (1 - \gamma_5) \psi_p(x)$$

$$J_{V,0} \text{ is vector current, } \Delta S = 0$$

$$J_{A,0} \text{ is axial vector current } \Delta S = 0$$

More generally we can write.

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$$\bar{J}_{\text{str}}^d = \bar{J}_{V,0}^d + \bar{J}_{A,0}^d + \bar{J}_{V,1}^d + \bar{J}_{A,1}^d$$

where last two terms give $|D_S| = 1$

Decay of $\bar{K}_0 \rightarrow \bar{K}_0$ discussed by Gell-Mann & Pais

in 1955.

Eigenstates of mass operators are eigenfunctions of CP but not of S.

Introduce $K_L \rightarrow K_S$ as linear combination of K_0, \bar{K}_0

then $K_L \rightarrow K_S$ are eigenstates of CP but not of S.

In decay S is not conserved for CP is

ex. K_0 produced by strong interaction is

a mixture of K_L, K_S — a beam will produce

K_S decays leaving K_L only — now K_L is a mixture

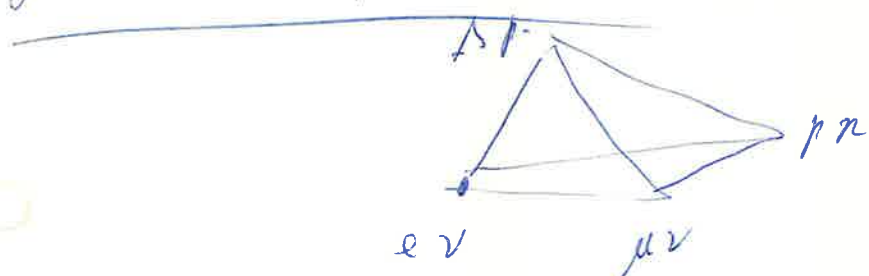
of K_0, \bar{K}_0 so \bar{K}_0 is regenerated — if \bar{K}_0 is

not observed by strong interaction, we are left with K_0 only

which regenerates K_S so decay of K_S is again

observable K_S decays into 2 pion state or 3 pion \pm
 K_L - - 3 pion decay. only

Gell-Mann Puppis tetrahedron



coupling between Δ^1 states & other states results
 a change in strangeness.

Decay can be leptonic or non-leptonic

remember. $\pi \equiv p\pi$ by strong interaction

and $\bar{\pi} \bar{\pi} \equiv K \equiv \Delta^1$ by strong interaction.

where mean is just a normalization of the correct
 vector coupling constant due to strong interaction
 effects

where m, τ

$$\bar{u}(p_2) \gamma^\mu (1-\gamma_5) u(p_1) \bar{u}(p_3) \gamma^\nu (1-\gamma_5) u(p_4) - 2 \gamma^\mu \gamma^\nu \bar{u}(p_2) \bar{u}(p_3)$$

FA also comes in ordinary β decay $p \rightarrow n + e + \bar{\nu}$
 where cross-section, including strong interaction

we write $J_{V,0}^\mu = 4\pi \gamma^\mu \psi_p + \dots$

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$$= \gamma^0 \psi - i \gamma^5 \psi.$$

J is the isotopic spin current.
which is conserved.

Hence we assume $J_{V,0}$ is a conserved current. CVC hypothesis

PCAC means partially conserved axial current
assumption is $\partial_\mu J_5^\mu \propto$ meson field operator.

J_5 is the axial current $J_{A,0}^{(q)}$

— basis of Goldberger-Treiman relation

calculate $F_A(q)$ & form factor

F_A is axial form factor — over a β decay of N^0

$$\langle \psi_p, J_{V,1}^\mu + J_{A,1}^\mu, \psi_N \rangle = \frac{1}{(2\pi)^3} \bar{u}(p_p) (F_V \gamma^\mu - F_A \gamma^\mu \gamma^5) u(p_N)$$

$$\rightarrow F_\pi \approx \frac{\sqrt{2}}{g} F_A(q)$$

F_π is form factor governing decay of π^+ -meson

We turn now to strong-interaction

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$J_{V,1}$ is of course in line with all baryon numbers.

Colloids suggest $J_{V,1}^d = \int d^4x \left(\bar{\psi}^{(4)} - i \bar{\psi}^{(5)} \right)$

where $F^{(4)}, F^{(5)}$ are octet currents.

So in general Colloids imply

$$J_V^d = \cos \theta \left(\int d^4x \left(\bar{\psi}^{(1)} - i \bar{\psi}^{(2)} \right) \right) + \sin \theta \left(\int d^4x \left(\bar{\psi}^{(4)} - i \bar{\psi}^{(5)} \right) \right)$$

$DS=0$

$DS=1$

i.e. spin current

$F^{(i)}(t) = \int_{x_0=t} d^3x \bar{\psi}^{(i)}(x)$ are generators of SU3

Octet currents obey

$$[\bar{\psi}^{(i)}(x), F^{(j)}] = i f_{ijk} \bar{\psi}^{(k)}(x)$$

i.e. have octet transformation properties.

Also assume $\partial_\mu \bar{\psi}^{(i)}(x) \propto \Phi_i(x)$
Percy Higgs

For J_A^d Casimir energy

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$$J_A^d = \cos \theta (F_5^{(1)d} - i F_5^{(2)d}) + \sin \theta (F_5^{(3)d} - i F_5^{(4)d})$$

Jell-Mann's current algebra

$$\text{Define } F_5^{(i)}(t) = \int_{x_0=t} d^3x F_5^{(i)0}(x)$$

$$\text{then } [F^{(i)}(t), F_5^{(j)}(t)] = i \delta_{ij} F_5^{(k)}(t) \quad (1)$$

since $F_5^{(i)}$ has octet transformation law.

we also have of course.

$$[F^{(i)}(t), F^{(j)}(t)] = i f_{ijk} F^{(k)}(t) \quad (2)$$

$$\text{What can we say about } [F_5^{(i)}(t), F_5^{(j)}(t)]$$
$$= i f_{ijk} F^{(k)}(t)$$

is algebra. assumed by Jell-Mann to close

leads to Adler-Warshawer relation

$$\frac{1}{F_A^2(0)} = 1 + \frac{2\pi^2}{(192)} \int_0^\infty \frac{d\nu'}{\nu'^2} (\sigma_{tot}^-(\nu') - \sigma_{tot}^+(\nu'))$$

cross-sections are for non-fermion

Analyticity of S-matrix

Dispersion relation etc¹

We start from unitarity of the S-matrix

$$S S^\dagger = S^\dagger S = 1$$

this expresses fact $|\psi_a^{\text{out}}\rangle = S |\psi_a^{\text{in}}\rangle$

$$|\psi_a^{\text{in}}\rangle = S |\psi_a^{\text{out}}\rangle$$

$$\text{then } \langle \psi_a^{\text{in}} | \psi_a^{\text{in}} \rangle = \langle \psi_a^{\text{out}} | S^\dagger S | \psi_a^{\text{out}} \rangle$$

a more general $\langle \psi_\beta^{\text{in}} | \psi_a^{\text{in}} \rangle = \langle \psi_\beta^{\text{out}} | S^\dagger S | \psi_a^{\text{out}} \rangle = S_{\beta a}$

show that $(S^\dagger S)_{\beta a} = \delta_{\beta a}$ in the out space.

$$\text{hence } S^\dagger S = 1$$

we write $S = \frac{1+R}{1+iR}$, gives

$$(1+R^\dagger)(1+R) = 1$$

$$\text{or } 1 + R^\dagger R + R + R^\dagger = 1$$

$$\text{or } -R^\dagger R = R + R^\dagger$$

if $R \Rightarrow \langle R \rangle$, then

$$R^\dagger R = i(R - R^\dagger) = 2i \text{Im} R$$

now under $(\psi_B^{\text{in}}, \psi_C^{\text{in}}) = -(2\pi)^4 i \delta(p_A - p_C) T(p, q)$

we find $i (T^*(p, q; p', q') - T(p', q', p, q))$
 $= -(2\pi)^4 \sum_n \delta(p_n - p - q) T^a(n, p', q')$
 $+ (n, p, q).$

for forward scattering $p' = p, q = q'$

$\Rightarrow 2 \text{Im } T(p, q, p, q) = |T(n, p, q)|^2.$

leads to $\text{Im } f(w, \theta) = \frac{q}{4\pi} \sigma_{\text{tot}}(w)$
optical theorem

the scattering amplitude

$f(w, \theta, \phi) = -\frac{16\pi^5 M}{w} T(p, q'; p, q)$

relates scattering amplitude to the T matrix

(cross-section is given in terms of f say.

$\left. \frac{d\sigma_{\text{el}}}{d\Omega} = \sum_{\text{spin}} |f(w, \theta, \phi)|^2 \right)$

this is the so-called optical theorem

3

But we can also derive generalized unitarity
conditions from the reduction formulas.

$$\text{Eikonal } (p'_1 q'_1 | R | p_1 q_1) + (p'_1 q'_1 | R^+ | p_1 q_1) = (1) \quad (1)$$

as a sum over states, reduces to optical
theorem in s-channel, but now gives
contributions in the t-channel, also no-pole
poles.

In general unitarity sum in optical theorem obtains
additional terms every time new physical process
needs threshold.

$$(1) \text{ can be written as } (p'_1 q'_1 | R | p_1 q_1) + (p_1 q_1 | R | p'_1 q'_1)^* \\ \propto i (T(p'_1 q'_1, p_1 q_1) - T^*(p_1 q_1, p'_1 q'_1))$$

which is then expressed by reduction formulas.

publish new concert
to report new
on down way
of
publish new concert
to report new
on down way
of
publish new concert
to report new
on down way
of

Crossing Symmetry

s-channel

$$s > 0$$

$$t < 0$$

$$\sqrt{s} = \text{c.d.m. energy} \quad u < 0$$

u-channel

cross $3 \leftrightarrow 1$ to give

$$A_2 + \bar{A}_3 = \bar{A}_1 + A_4$$

$$\text{nucl.} + \text{anti neutr} = \text{anti neutr} + \text{nucl.}$$

$$s = (p_1 + p_2)^2$$

$$t = (p_3 - p_1)^2$$

$$u = (p_4 - p_1)^2$$

$$A_1 + A_2 = A_3 + A_4$$

neu nucl = neu + nucl.

$$\bar{A}_3 \quad A_4$$

$$\bar{A}_1 \quad A_2$$

$$p_3 \Rightarrow -p_1$$

$$s \rightarrow (p_2 - p_3)^2 = u$$

$$p_1 \rightarrow -p_3$$

$$t \rightarrow (p_3 - p_1)^2 = t$$

$$u \rightarrow (p_4 + p_3)^2 = (p_1 + p_2)^2 = s$$

for $\sqrt{u} > \text{c.d.m. energy}$

$$u > 0, \quad s < 0, \quad t < 0$$

t-channel

cross $4 \leftrightarrow 1$

$$A_2 + \bar{A}_4 = \bar{A}_1 + A_3$$

$$\text{nucl.} + \text{anti neutr} \rightarrow \text{neu} + \text{anti neutr}$$

$$\bar{A}_3 \quad \bar{A}_4$$

$$\bar{A}_1 \quad A_2$$

$\sqrt{t} = \text{c.d.m. energy}$

$$p_4 \Rightarrow -p_1$$

$$s \rightarrow (p_2 - p_4)^2 = t$$

$$p_1 \rightarrow -p_4$$

$$t \rightarrow (p_3 + p_4)^2 = (p_1 + p_2)^2 = s$$

$$u \rightarrow (p_4 - p_1)^2 = u$$

So u-channel exchanges s and u, leaves t unchanged.

t-channel exchanges s & t, leaves u unchanged

T.C.P. ensure of $A_1 + A_2 = A_3 + A_4$

ie. $\{2$

$$i \quad \bar{A}_3 + \bar{A}_4 = \bar{A}_1 + \bar{A}_2 \quad \left. \begin{array}{l} \text{mean } p_3 \rightarrow -p_1, p_4 \rightarrow p_2 \\ p_1 \rightarrow -p_3, p_2 \rightarrow -p_4 \end{array} \right\} \begin{array}{l} s \rightarrow s \\ t \rightarrow t \\ u \rightarrow u \end{array}$$

similar T.C.P. ensures of u -channel ^{which is invariant}

$$A_1 + \bar{A}_4 = \bar{A}_2 + A_3$$

and T.C.P. ensure of t -channel

$$A_1 + A_3 = \bar{A}_2 + A_4$$

Bound
T.C.P. theorem

for spinless bosons amplitudes for s, u, t channels are all identical, both to the notion of crossing symmetry in $\phi(s, t, u)$

we can use $-p_3, -p_4$ for u -momenta of emerging particles, which gives symmetrical form

$$t = (p_1 + p_3)^2$$

$$u = (p_1 + p_4)^2$$

$$s = (p_1 + p_2)^2$$

$$p_1 + p_2 = p_3 + p_4$$

$$\text{Note that } t+u+s = 2 + 2p_1 \cdot p_2 + 2 - 2p_1 \cdot p_3 + 2 - 2p_1 \cdot p_4$$

$$\therefore s+t+u = 6 + 2p_1 \cdot p_2 - 2p_1 \cdot (p_3 + p_4) = 4m^2$$

Dispersion relations

we consider two particle scattering as an example

$F(s, t)$ say describes $\langle p_3, p_4 | T | p_1, p_2 \rangle$

$$s = (p_1 + p_2)^2 \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2.$$

consider fixed value of t , now go to u -channel
thresholds will occur at $u = 4m^2 \quad 9m^2 \quad 16m^2 \dots$

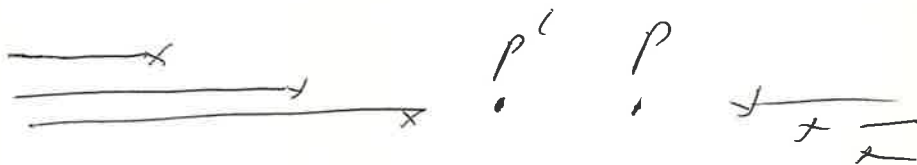
$$\text{corresponds to } s = 4m^2 - t_0 - u$$

$$= -t_0, -t_0 - 5m^2, \dots$$

so in complex s -plane, for fixed $t = t_0$, singularities

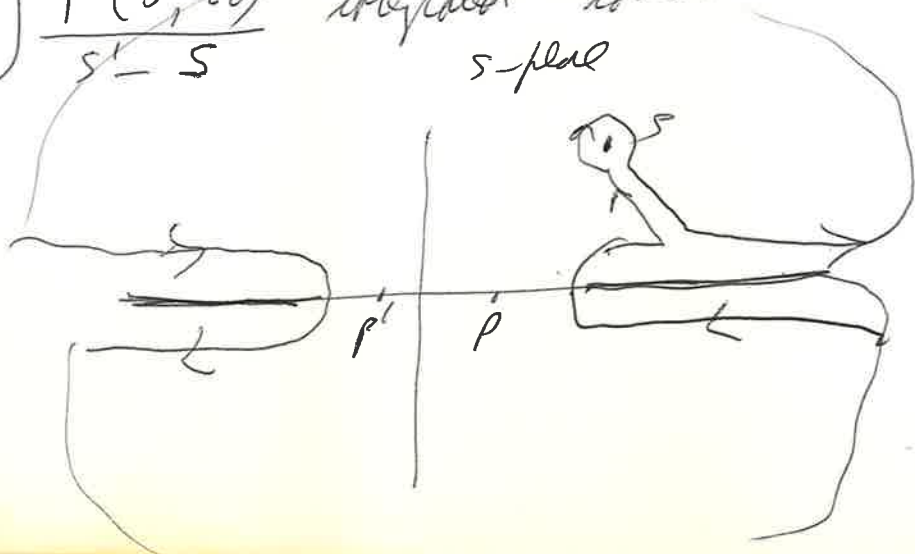
are as shown.

poles at $s = m^2$
and $s = 3m^2 - t_0$



To obtain dispersion relation for $F(s, t)$

consider $\frac{1}{2\pi i} \int \frac{f(s', t_0)}{s' - s} ds'$ integrated round contour shown in s -plane



$$\text{rest is } -f(s, t_0) + \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{ds' F(s', t_0)}{s' - s} \quad \underline{\quad}$$

$$+ \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{ds' F(s', t_0)}{s' - s} = \frac{2\pi i}{2\pi i} \sum \frac{A_i}{s_i - s}$$

where A_i is a residue of $F(s', t_0)$ at pole P_i

$$\text{Hence } f(s, t_0) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{ds'}{s' - s} F(s', t_0) \quad (1)$$

$$+ \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{ds'}{s' - s} F(s', t_0) + \sum \frac{A_i}{s_i - s} = P$$

poles P are given by

$$P = \frac{g_s^2}{s - m^2} + \frac{g_t^2}{s + t_0 - 3m^2}$$

$g_s > g_t$ are constants

F_s is discontinuity in F across R.H. cut. - disc. $F = 2i \operatorname{Im} F$
if F is real between cuts on real axis, no scattering
reflection principle applies

$$\text{disc } F = F(x+i\epsilon) - F(x-i\epsilon) = F(x+i\epsilon) - F^*(x+i\epsilon) = 2i \operatorname{Im} F$$

and F is $\lim_{\epsilon \rightarrow 0} F(x+i\epsilon)$ for ϵ small in s -channel.

F is discontinuous in F in the u -channel. 3

Since in this case. physical limit in u -channel.

is $\lim_{\epsilon \rightarrow 0} F(s-i\epsilon, t_0)$ for $s < 0$.

Making use of $s' + t' + u' = 4m^2$

we can write.

from $s + t_0 = 3m^2$
 $m^2 - u$

$$F(s, t, u) = \frac{q_s^2}{s-m^2} + \frac{q_u^2}{u-m^2} + \frac{1}{2\pi i} \int_{4m^2}^{\infty} ds' \frac{F(s', u', t)}{s'-s}$$

$$+ \frac{1}{2\pi i} \int_{4m^2}^{\infty} du' \frac{F_u(s', u', t)}{u'-u} \quad (2)$$

(q_u^2 is dropped
sign on 1^{st} term)

Since $u' = 4m^2 - s' - t$ in 2nd integral

$$ds' = -du', \quad s' = -t, \quad u' = 4m^2$$

$$s' = -\infty, \quad u' = \infty.$$

$$s' - s = 4m^2 - u' - t - (4m^2 - u - t) = -(u' - u)$$

(2) plus relation to the Parseval rep. abet

lets the form $F(s, t, u) = P + \frac{1}{\pi^2} \iint \frac{P_{st}(s', t')}{(s'-s)(t'-t)} ds' dt'$

$$+ \frac{1}{\pi^2} \iint \frac{P_{tu}(t', u')}{(t'-t)(u'-u)} dt' du' + \frac{1}{\pi^2} \iint \frac{P_{us}(s', u')}{(u'-u)(s'-s)} ds' du' \quad (3)$$

for double dispersion relation. Parseval's theorem

Remember $s' + t' + u' = 4m^2$.

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By integrating over t' , we can recast the single denominator relation, if we consider $s + t + u = 4m^2$ and keep t , fixed as our denominator.

So s and u are related by $s = 4m^2 - t - u$.

To do this we can integrate 1st & 2nd terms in (3) directly and then compare this term

$$\text{or follow } \frac{1}{\pi^2} \iint \frac{P_{us}(s', u') ds' du'}{(u' - u)(s' - s)}$$

$$\begin{aligned} \text{Since } \frac{1}{(u' - u)(s' - s)} &= -\frac{1}{(t' - t)(s' - s)} - \frac{1}{(t' - t)(u' - u)} \\ &= \frac{-(u' - u) - (s' - s)}{(t' - t)(s' - s)(u' - u)} = \frac{-(u' + s') + (u + s)}{(t' - t)(s' - s)(u' - u)} \\ &= \frac{-(4m^2 - t) + (4m^2 - t)}{(t' - t)(s' - s)(u' - u)} \\ &= \frac{1}{(s' - s)(u' - u)} \end{aligned}$$

$$\therefore \frac{1}{\pi^2} \iint = -\frac{1}{\pi^2} \iint du' ds' \frac{P_{us}(u', s')}{(t' - t)(s' - s)} - \frac{1}{\pi^2} \iint du' ds' \frac{P_{us}(u', s')}{(t' - t)(u' - u)}$$

We can now do the integrations to reach the right needed denominator relations. such as (2)

p. 7. F_S on \mathcal{B} is given by

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$$\frac{1}{2\pi i} F_S = \frac{1}{\pi^2} \int_{4m^2}^{\infty} dt' \frac{C_{S+}(s', t')}{t' - t}$$

never was
clear from Eqn. \rightarrow

$$\frac{1}{\pi^2} \int_{-\infty}^{-s'} dt' \frac{P_{us}(4m^2 - s' - t', s')}{t' - t}$$

where we have gone from u' to t' on variable
of integration

$$t' = 4m^2 - u' - s' \quad \frac{dt'}{du} = -1$$

$$u' = \infty \quad \text{then } t' = -\infty$$

$$u' = 4m^2 \quad t' = -s'$$

Subtraction we have assume $f(s, t_0) \rightarrow 0$ (along \overline{s})

a) integral over circle vanishes

if the γ is not the circle, replace

$$G(s, t_0) = \frac{F}{(s-s_1) \dots (s-s_n)} \quad \text{with } s_1, \dots, s_n \text{ are poles}$$

substituting the residues.

then apply the theorem related to the integral formula to get

$$F(s, u, t_0) = \phi(u) + \frac{q_s^2}{s-m^2} + \frac{q_u^2}{u-m^2} + \frac{(s-s_1) \dots (s-s_n)}{2\pi i}$$

$$+ \left\{ \int_{4m^2}^{\infty} \frac{ds' F(s', u', t_0)}{(s'-s)(s'-s_1) \dots (s'-s_n)} \right.$$

$$+ \left. \int_{4m^2}^{\infty} \frac{du' F_u(s', u', t_0)}{(u'-u)(u'-u_1) \dots (u'-u_n)} \right\}$$

where $t_0 = t_n - u_n - 4m^2$ as defined.

$$u_n = 4m^2 - t_0 - s_i$$

and $s' + u' + t_0 = 4m^2$ as usual.

Φ is a polynomial of degree $(N-1)$ in s .

derived from identity residues at poles s_1, \dots, s_n - 6

then
$$\frac{G(s')}{s' - s} = \frac{F(s')}{(s' - s_1) \cdot (s' - s_2) \cdot (s' - s_n)}$$

Residue at s_1 is
$$\frac{F(s_1)}{(s_1 - s) (s_1 - s_2) \dots (s_1 - s_n)}$$

or $(s - s_1) \cdot (s - s_n) \times$ residue

$$= (s - s_2) \cdot (s - s_n) \times \text{residue}$$

which is poly. of degree $(n-1)$ so

degree N coefficients of $\Phi(N)$ are.

called subtraction constants & are

subtracted rather than added to have

N subtractions.

In Nordberg's up. $P_{s+}(s', t)$ doesn't
obey dispersion sum as $s \rightarrow \infty$ & doesn't
satisfy.

$q \ll p(u)$ is not used for $u \leq 0$
 then $f, p(u) \equiv \text{dure. } f$
 $\equiv \text{L.H.S. of condn}$
 condn

Is that we can use the recurrent
 condn in its sparse form
 case $(1/2) = (2/1)$ is satisfied
 in the condn $f(u)$ need a to
 when $f(u) = 29$ need more values than as -

Further discussion follows can be made

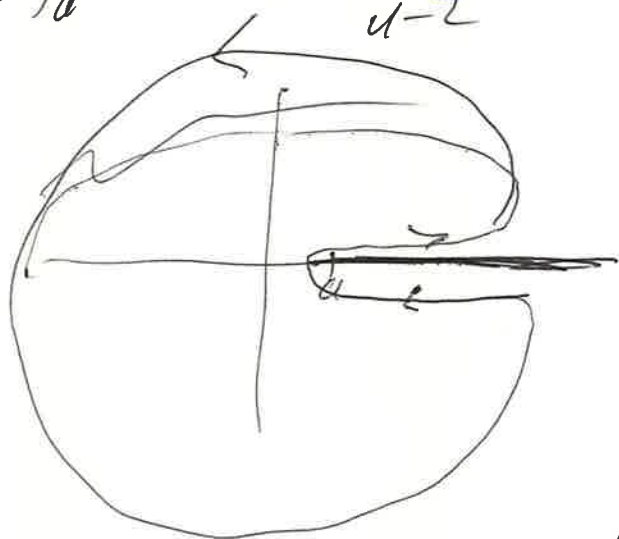
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e.g. Edm. (1967) gives slightly smaller.

$$f(z) = \frac{1}{2\pi i} \int_C \frac{f(u) du}{u-z}$$

- assume 1.) $f(u)$ no singularities except those at $u=a$ and $u=b$
 2.) $f(u) \rightarrow 0$ as $u \rightarrow \infty$.

then $f(z) = \frac{1}{2\pi i} \int_0^\infty du \frac{f(u+i\epsilon) - f(u-i\epsilon)}{u-z}$ with $\epsilon \rightarrow +0$



If 3.) $f(u)$ is real for $u \leq a$, Schwarz's reflection principle gives $f(z^*) = f^*(z)$

$$f(z) = \frac{1}{\pi} \int_a^\infty du \frac{f_1(u)}{u-z}$$

where $f_1(u) = \frac{1}{2i} (f(u+i\epsilon) - f(u-i\epsilon)) = \text{Im } f(u)$
 is assumed finite

from $f(z) = \frac{1}{\pi} \int_0^\infty du \frac{\text{Im } f(u)}{u-z}$

we can obtain $\text{Re } f(x) = \frac{1}{\pi} P \int_0^\infty du \frac{\text{Im } f(u)}{u-x}$

where $x \rightarrow x+i\epsilon$.

and $\text{Im } f(x) = \frac{1}{\pi} \text{Im } f(x)$ as required for consistency.

Now we give example of forward scattering of pions by pions.

$f(q)$ is forward scattering amplitude as function of pion mass q .

Let $f(q)$ is analytic in upper half-plane.

Let $f(q)$



$f(q) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} dq' \frac{f(q')}{q' - q}$

q off-shell and we can do.

It follows that

2

$$F(q) = \frac{1}{\pi i} P \int_{-\infty}^{\infty} dq' \frac{F(q')}{q' - q} \quad \left(\leftarrow \text{cf. Gario p. 350} \right)$$

$$\text{then } \operatorname{Re} F(q) = \frac{1}{\pi} P \int_{-\infty}^{\infty} dq' \frac{\operatorname{Im} F(q')}{q' - q}$$

$$\text{using } \operatorname{Im} F(-q') = -\operatorname{Im} F(q')$$

$$\text{we can write } \operatorname{Re} F(q) = \frac{1}{\pi} P \int_0^{\infty} 2q' dq' \frac{\operatorname{Im} F(q')}{q'^2 - q^2}$$

For use in the part entitled
of a dispersion relation to be derived
by Jell-Mann, Goldberger &
Horn 1955.

From the fact on to the Padé table
ref. to obtain forward scattering differences
as from - nuclear scattering, our other
reference formula to help.

$$\begin{aligned}
 & \frac{e^{-i\hbar\omega} - e^{-i\hbar\omega_0}}{e^{-i\hbar\omega} - e^{-i\hbar\omega_0}} + (e^{-i\hbar\omega} - e^{-i\hbar\omega_0}) \\
 & = -e^{-i\hbar\omega} + e^{-i\hbar\omega_0} \\
 & = -e^{-i\hbar\omega} + e^{-i\hbar\omega_0} + (e^{-i\hbar\omega} - e^{-i\hbar\omega_0})
 \end{aligned}$$

Partial waves, dispersion relations: Page 100

we adopt the partial wave expansion

$$f(s, t) = \sum_{n=0}^{\infty} (2n+1) a_n(s) P_n(\cos \theta) \quad (1)$$

where $a_n = \frac{1}{2iR} (e^{2i\eta_n} - 1) = \frac{e^{i\eta_n} \sin \eta_n}{R}$

where $R^2 \propto E$, energy of particle.
in potential scattering.

In general, $e^{2i\eta_n}$ is partial-wave coeff. for S-matrix

$(e^{2i\eta_n} - 1) \propto$ coeff. for scattering amplitude. $S = 1 + f$ on shell.

Thus $\psi \propto e^{ikr} + f e^{ikr}$
 $= e^{-ikr} + (1+f) e^{ikr} = e^{-ikr} + S e^{ikr}$

ordered
 $e^{-ikr} - S e^{ikr}$
 for $\theta=0$, cf. 11.17
 p. 391

shows general relationship between f & S .

Thus from (1) $a_n = \frac{1}{2} \int_{-1}^1 d(\cos \theta) \tilde{f}(s, \cos \theta) P_n(\cos \theta)$

$\left[\psi \propto -e^{-ikr} + S e^{ikr} = -(e^{-ikr} - S e^{ikr}) \right]$

Since $\int_{-1}^1 P_n(u) du = \frac{2}{2n+1}$ and $\tilde{f}(s, \cos \theta) = f(s, \theta)$

from any unitary condition for elastic scattering $\frac{2}{\eta_e}$
 yields. in terms of f or $\frac{1}{f}$ notation we obtain.

$$\text{we obtain } i(a_e^\dagger - a_e) = 2|a_e|^2.$$

$$\text{where } a_e = \frac{e^{2i\eta_e} - 1}{2i\eta_e} = e^{i\eta_e} \frac{(e^{i\eta_e} - e^{-i\eta_e})}{2i\eta_e}$$

η_e is real. with \sin elastic phase η_e is just a real constant phase.

$$= e^{i\eta_e} \frac{\sin \eta_e}{\eta_e}.$$

If inelastic processes do occur, we have

$$i(a_e^\dagger - a_e) = 2|a_e|^2 + 2P_e.$$

$$S_e(\text{elastic}) = 1 + 2i a_e \quad \text{as before.}$$

$$\text{we note that } P_e = \text{Im } a_e - |a_e|^2. \quad \text{--- (1)}$$

$$\text{we can infer } 0 \leq P_e \leq 1/4.$$

$$\text{Writing } a_e(w) = \frac{\eta_e e^{2i\eta_e} - 1}{2i\eta_e} \quad (\text{using notation})$$

$$\text{we can choose } \eta_e \text{ real, and use } 0 \leq \eta_e \leq 1$$

$$\text{for } P_e = \frac{1}{4}(1 - \eta_e^2)$$

It follows from eq. (1)

$$a_l = \frac{(\eta_l \cos 2\delta_l - 1) + i \eta_l \sin 2\delta_l}{2i}$$

$$\begin{aligned} P_l &= -\frac{1}{2} (\eta_l \cos 2\delta_l - 1) - \frac{1}{4} [(\eta_l \cos 2\delta_l - 1)^2 + \eta_l^2 \sin^2 2\delta_l] \\ &= \frac{1}{2} - \frac{1}{2} \eta_l \cos 2\delta_l - \frac{1}{4} (\eta_l^2 + 1 - 2\eta_l \cos 2\delta_l) \\ &= \frac{1}{4} (1 - \eta_l^2) \text{ as stated.} \end{aligned}$$

We can define 3 partial cross-sections: —

$\sigma^l(\text{elastic})$ $\sigma^l(\text{absorptive})$ $\sigma^l(\text{total})$, say

$$\sigma(\text{total}) = \sum_{l=0}^{\infty} \sigma^l(\text{total})$$

σ_{total} is still given by the optical theorem.

$$\propto \sum_{l=0}^{\infty} (2l+1) \operatorname{Im} a_l$$

where we find

$$\left. \begin{aligned} \sigma^l(\text{total}) &= \frac{(2l+1)\pi}{k^2} (2 - 2\eta_l \cos 2\delta_l) \\ \sigma^l(\text{elastic}) &= \frac{(2l+1)\pi}{k^2} (1 - 2\eta_l \cos 2\delta_l + \eta_l^2) \\ \sigma^l(\text{absorptive}) &= \frac{(2l+1)\pi}{k^2} (1 - \eta_l^2) \end{aligned} \right\} \begin{matrix} (1) \\ (2) \\ (3) \end{matrix}$$

which (1) = (2) + (3).

Optical theorem from Fermi-Hellmuth formula 4

$$\text{Start - then } f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (e^{2i\delta_l} - 1) P_l(\cos\theta)$$

which is derived from plane wave expansion.

$$e^{ikr} - e^{ikr} = \sum_{l=0}^{\infty} (2l+1) i^{-l} j_l(kr) P_l(\cos\theta)$$

$$j_l \sim (kr)^{-1} \sin(kr - \frac{1}{2}l\pi) \quad j_l = \sqrt{\frac{\pi}{2kr}} J_{l+1/2}(kr)$$

$$\text{then } \text{Im } f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l \quad \text{or } f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin^2 \delta_l P_l(\cos\theta)$$

$$\text{But } \sigma_{\text{tot}} = \int |f(\theta)|^2 d\Omega = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l$$

$$= \frac{4\pi}{k} \text{Im } f(\theta)$$

But is still true for inelastic scattering

In presence of inelastic scattering we can write:

$$\sigma_{\text{inc}} \sim \pi^{-1} C_e e^{ikr} P_l(\cos\theta) \quad C_e = e^{2i\delta_l}$$

$$\sigma_e(\text{elastic}) \propto |C_e|^2$$

$$\text{or } \sigma_e(\text{tot}) \propto \sum C_e \quad , \quad \text{from optical theorem}$$

→ change in definition
of 10 sec

N5

then we find $\sigma_0(\text{value}) = \sigma(\text{total}) - \sigma(\text{value})^5$

alternatively we can compute $\sigma(\text{value})$ by
 calculating with observed value from the log
 then using $\psi = \psi_{\text{true}}^e + \psi_{\text{red}}^e$

then $N = -\frac{c\hbar}{2\pi} \int \left(\psi^* \frac{\partial \psi}{\partial r} - \frac{\partial \psi^*}{\partial r} \cdot \psi \right) dV$
 or here just

$$\sigma_{\text{value}} = \frac{4\pi}{2l+1} \left\{ \frac{i}{2\hbar} (2l+1) (c_0^+ - c_0) - |c_0|^2 \right\}$$

↑ (total) for ↑ polar.
 optical down

another way of stating problem is to use a
 complex plane shift.

i.e. let $c_0 = \frac{1}{2i\hbar} (e^{2i\eta_0} - 1)$ with $\eta_0 = \lambda_0 + i\eta_0$
 earlier

then $c_0 = \frac{1}{2i\hbar} (2l+1) (e^{2i\eta_0} - 1)$

(so in previous problem we use that $\eta_0 = e^{-2\mu_0}$)
 then get (1) (2) (3) a bound.

6

e.g. $\sigma_{\text{scatter}} = \frac{(2\ell+1)\pi}{k^2} (1 - |\eta_\ell|^2)$

$$= \frac{(2\ell+1)\pi}{k^2} (1 - e^{-4\mu_\ell})$$
$$= \frac{(2\ell+1)\pi}{k^2} e^{-2\mu_\ell} (e^{2\mu_\ell} - e^{-2\mu_\ell})$$

$$= \frac{2\pi}{k^2} (2\ell+1) e^{-2\mu_\ell} \sinh 2\mu_\ell$$

or μ_ℓ in $\sqrt{\ell+1/2} \sqrt{R}$
Ellen p. 151

6

we have $f_e = \frac{e^{2i\eta_e} - 1}{2i}$

Residue Theorem

$P_e = 1 + f_e = 1 + 2if_e = e^{2i\eta_e}$

write $R_e = \tan \eta_e$

$S_e = e^{2i\eta_e}$

Scattering
relation

$\frac{1}{2} \propto e^{-i\eta_e} S_e e^{i\eta_e}$

relates R_e & S_e

and $\sin \eta_e + \tan \eta_e \cos \eta_e$

a simple case of

$= \sin \eta_e + R_e \cos \eta_e$

R_e partial scattering

\hookrightarrow Resonance
relation.

then $S = \frac{1 + iR_e}{1 - iR_e}$

in the Resonance, S is unitary.

$S_e = \frac{1 + i \tan \eta_e}{1 - i \tan \eta_e} = \frac{(1 + i \tan \eta_e)(1 + i \tan \eta_e)}{1 + \tan^2 \eta_e}$

$= \frac{1 - \tan^2 \eta_e}{1 + \tan^2 \eta_e} + 2i \frac{\tan \eta_e}{1 + \tan^2 \eta_e}$

$= \cos 2\eta_e + 2i \frac{\sin \eta_e}{\cos \eta_e \cos^2 \eta_e}$

$= \cos 2\eta_e + 2i \sin \eta_e \cos \eta_e$

$= \cos 2\eta_e + 2i \sin \eta_e \cos \eta_e = \cos 2\eta_e + i \sin 2\eta_e = e^{2i\eta_e}$

\hookrightarrow de Nouvel's theorem

These results can
be generalized for
multi-channel
scattering

(cf. P. 11)

According to M & M p. 138.

General Representation of
partial wave S-matrix

In general we write

$$G_0(0) = 0 \quad G_l \sim A(k) \sin(kr - \frac{1}{2}l\pi + \eta_l)$$

or alternatively $f_l(\pm) \sim e^{\mp ikr} + \frac{1}{2}ie^{i\eta_l}$

for which $A(k) = e^{i\eta_l}$

$$\begin{aligned} G_l &\rightarrow e^{i\eta_l} \sin(kr - \frac{1}{2}l\pi + \eta_l) \\ &\sim e^{i\eta_l} \frac{e^{i(kr - \frac{1}{2}l\pi + \eta_l)} - e^{-i(kr - \frac{1}{2}l\pi + \eta_l)}}{2i} \end{aligned} \quad (1)$$

also $G_l = \frac{1}{2}i \left[b(+)(-1)^{l+1} S_l b(-) \right] \quad S_l = e^{2i\eta_l}$

$$\sim \frac{1}{2}i \left[e^{-ikr + \frac{1}{2}i\eta_l} + (-1)^{l+1} S_l e^{ikr + \frac{1}{2}i\eta_l} \right] \quad (2)$$

① for $\frac{1}{2}i e^{-ikr + \frac{1}{2}i\eta_l} - \frac{2i\eta_l}{e} e^{ikr - \frac{1}{2}i\eta_l}$

As we require for ② to agree

$$(-1)^{l+1} e^{\frac{1}{2}i\eta_l} = -e^{-\frac{1}{2}i\eta_l}$$

$$\text{or } (-1)^{l+1} = -e^{-i\eta_l} = (-1)^l + (-1) = (-1)^{l+1}, \text{ which is correct.}$$

for n even

$$\psi_e \propto e^{-ikr} + (-1)^{l+1} e^{i(kr - l\pi)} e^{ikr}$$

for $l=0$

$$\psi \propto e^{-ikr} - e^{i(kr - l\pi)} e^{ikr}$$

So same for odd l

$$\psi_e \propto e^{-ikr} + S_0 e^{i(kr - l\pi)}$$

$$\text{for even } l \text{ then } \psi_e \propto e^{-ikr} - S_e e^{i(kr - l\pi)}$$

General definition has been given by Kohn
which involves both terms.

$$\text{write } G \propto (A e^{-i(kr - \frac{1}{2}l\pi)} + B e^{i(kr - \frac{1}{2}l\pi)})$$

then $B = S A$ defines S - matrix

$$\text{so } G \propto A \left(e^{-i(kr - \frac{1}{2}l\pi)} - S_0 e^{i(kr - \frac{1}{2}l\pi)} \right) \quad (2)$$

as general definition

Notice change in sign of $e^{\frac{i}{2}l\pi}$ between A & B (even)

P.T.O

Stimuli per R. action, alle taker

perdece

$$C_2 (C_{2m}(R - \frac{1}{2}en) + D \cos(h - \frac{1}{2}en))$$

per $D = RC$ depen Rorden Nulien R

to in low of R

$$C_2 (C_{2m}(R - \frac{1}{2}en) + R_{cos}(h - \frac{1}{2}en)) \quad (3)$$

Nulien + sign in (3) carload
make - per in (3)

Partial Wave Dispersion Relations

1

Derived from Randlestam Representations
multiplied by $P_\ell(\cos\theta)$ & integrated over $\cos\theta$ from 0 to π — $\cos\theta$ expressed in terms
of s and t , leads to cuts in
the partial wave amplitudes, partly derived from integration
this in turn leads to dispersion relations for $a_\ell(s)$
in the complex variable s .

On R.H. cut $\text{disc. } a_\ell(s)$ is $2i \text{Im } a_\ell(s)$ on physical sheet
 $\propto |a_\ell(s)|^2$

on L.H. cut we do not know what $\text{disc. } a_\ell(s)$ is
at all (no longer simply related to crossed
partial wave amplitudes, since unitarity is now subtle,
but crossing is complicated)

L.H. $\text{disc } a_\ell(s)$ plays role of potential

L.H. cut is referred to as the non-physical cut

Thus we start from. for pion-pion scattering

$$t = -2q^2(1 - \cos\theta) \quad s = 4(q^2 + m_\pi^2)^2$$

$$\cos\theta = \left(1 + \frac{2t}{s - 4m_\pi^2}\right)$$

we write for scattering amplitude

$$T = \frac{1}{\pi^2} \int_{4m_\pi^2}^{\infty} ds \int_{-1}^1 dy \, C(x, y) \\ + \left[\frac{1}{(x-s)(y-t)} + \frac{1}{(x-t)(y-u)} + \frac{1}{(x-u)(y-s)} \right]$$

then express t, u in terms of q^2 & θ
to get for partial-wave amplitude

$$b_\ell(u) \propto \int_{-1}^1 dy \, C(x, y) (t+t')^\ell \\ + \left(\frac{1}{x-s} + \frac{1}{x+y+4q^2} \right) Q_\ell \left(1 + \frac{y}{2q^2} \right)$$

where Q_ℓ is Legendre function of 2nd kind, defined
well at $\cos\theta = -1$ to 1

Note on
Protonium, neutron atoms, bound states

see in neutron capture at $k = a - i2$
 leads to $E \approx k^2 = a^2 + v^2 - 2av$ ⁶⁷⁰

$-a \pm i \rightarrow$ $-a \pm i$ \times Resonance term

averaged to decay of unstable particle

Resonance term $k = a - i2$ if v is small

$| \lambda \text{ dependence in } e^{ikr} = e^{ia} + \text{oscillatory term}$

same as for theory of d -decay

For $a \approx 0$ — λ dependence is 0 For k large

to neutron state —

For $k = 0$, $v < 0$, λ -dependence is 0 — true bound state —

Writing $t = \sqrt{s} f_e$ $v = q^2$, $t = t(v)$

3

then we obtain desired relation.

$$t_e = \frac{1}{\sigma} \int_0^\infty dv' \frac{\rho_e(v') / |t_e(v)|^2}{v' - v - i\epsilon} + \frac{1}{\pi} \int_{-\infty}^{-\infty} dv' \frac{\phi_e(v')}{v' - v - i\epsilon}$$

A typical example

where $\rho_e(t_e) = \rho_e(v) / |t_e(v)|^2$ can

p.v. cal.

Effective range approx write $v^e \rho_e \frac{1}{t_e(v)} \approx q_e + b_e v$

leads to $\frac{q^{2e+1}}{\sqrt{s}}$ cot $S_e(v) \approx q_e + b_e v$

$$\text{and } \rho_e(v) = \frac{1}{q \cot S_e(v) - i q} \quad \left(= \frac{e^{i S_e} \sin S_e}{q} \right)$$

ideal \rightarrow
constant limit

effective range expansion is of $q \cot S_e$ a power of $q(v)$

leads to B-T formula as to

resonance from near zero of $q \cot S_e$

a bound state near zero of $q \cot S_e - i q$.

we look to study of resonance, bound states
near $k^2 = 0$

① on 4 shell of Φ a brown 15
— the rest of the material, which
is N/D nested S. 9.

Brothstep Affair ~~reserves in C~~

Φ deems from nothing is curved
down. Φ reserves in Φ -dune
is enclosed to "brothstep" Φ , then
will the lead to some reserves
in S-dune — Φ for material

2 reserves enclosed C-term, leads
to C-term reserves e.g.

① $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$ (L'Hôpital's Rule)

Residue of $\frac{f(z)}{g(z)}$ at z_0 is $\lim_{z \rightarrow z_0} (z - z_0) \frac{f(z)}{g(z)}$

Let $z_0 = 0$ and $f(z) = \frac{1}{z}$, $g(z) = \frac{1}{z^2}$

Then $\lim_{z \rightarrow 0} (z - 0) \frac{f(z)}{g(z)} = \lim_{z \rightarrow 0} z \cdot \frac{1/z}{1/z^2} = \lim_{z \rightarrow 0} z \cdot z = \lim_{z \rightarrow 0} z^2 = 0$

Residue of $\frac{f(z)}{g(z)}$ at z_0 is $\lim_{z \rightarrow z_0} (z - z_0) \frac{f(z)}{g(z)}$

Let $z_0 = 0$ and $f(z) = \frac{1}{z}$, $g(z) = \frac{1}{z^2}$

Then $\lim_{z \rightarrow 0} (z - 0) \frac{f(z)}{g(z)} = \lim_{z \rightarrow 0} z \cdot \frac{1/z}{1/z^2} = \lim_{z \rightarrow 0} z \cdot z = \lim_{z \rightarrow 0} z^2 = 0$

Regge Poles.

1

We start from scattering amplitude in form

$$f(u, \cos \theta) = \sum_{l=0}^{\infty} (2l+1) a_l(u) P_l(\cos \theta) \quad (1)$$

and seek, first of all, an interpolating

function for $a_l(s)$

\sqrt{s} is large in
w in p.d.m.
frame

$$a(l, s) = a_l(s) \quad l=0, 1, 2, \dots$$

uniqueness of $a(l, s)$ guaranteed by
 Carlson's theorem (assuming $a(l, s)$ vanishes
 for large l so $\lim_{l \rightarrow \infty} a(l, s) = 0$)

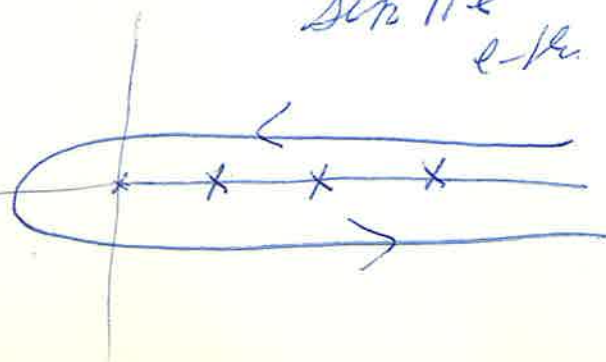
This expresses Σ as an integral over all l along
an appropriate contour, by introducing cosine πl
as a meromorphic function with residue $\frac{(-1)^l}{\pi}$
at poles at $l=0, 1, 2, \dots$

$$\text{Hence } f(u, \cos \theta) = \frac{1}{2i} \int_C dl \cdot (2l+1) a(l, s) \frac{P_l(\cos \theta)}{\sin \pi l}$$

C is contour like

path of integral at $l=0, 1, 2, \dots$

poles of $a(l, s)$ at Regge poles.



Backward integral is also
 line $\rho = -\frac{1}{2}$
 Let's assume that a unit of $\rho = -\frac{1}{2}$
 the function $a(\rho, s)$ is not included
 path.

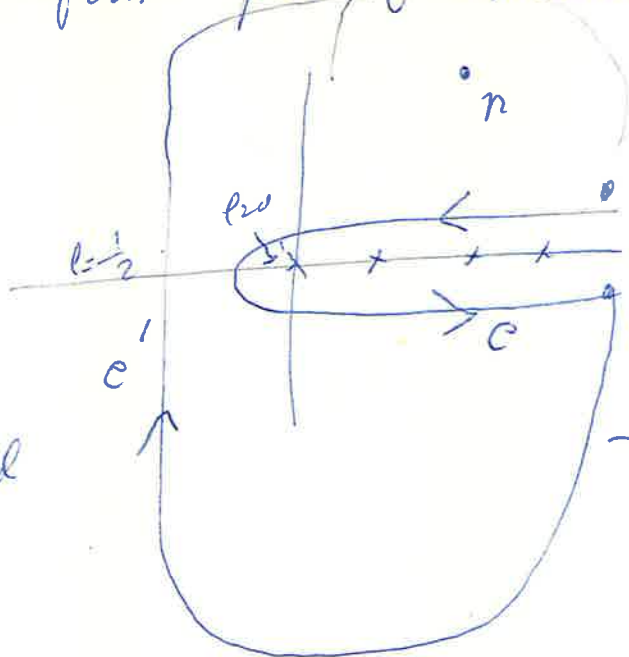
Watson - Sommerfeld Transform:

2

we now deform path of integration, des:-

l-plane.

Note $s = 4(m^2 + q^2)$
 for $q^2 = 0$ we have
 $s = 4m^2$
 q is momentum
 in c.d.w. plane
 \sqrt{s} is energy in s-channel
 in c.d.w. plane.



$q^2 > 0$ all
 poles in complex
 plane - poles
 on real axis only
 for $q^2 < 0$

$$\text{Then } \frac{1}{2\pi i} \int_{C'} + \frac{1}{2\pi i} \int_C = -\pi \sum_n d_n(s) \quad d_n \text{ is residue at } n^{\text{th}} \text{ Regge pole.}$$

if n' is Regge pole on real axis

$$\text{Then } \pi \sum_{n'} d_{n'}(s) + F - \frac{1}{2\pi i} \int_{C'} = -\pi \sum_n d_n(s)$$

$$\begin{aligned} n \quad f &= -\pi \sum_n d_n(s) - \pi \sum_{n'} d_{n'}(s) - \frac{1}{2\pi i} \int_{C'} \\ &= -\pi \sum_n d_n(s) - \frac{1}{2\pi i} \int_{C'} \end{aligned} \quad \text{--- (1)}$$

where n now includes all the Regge poles

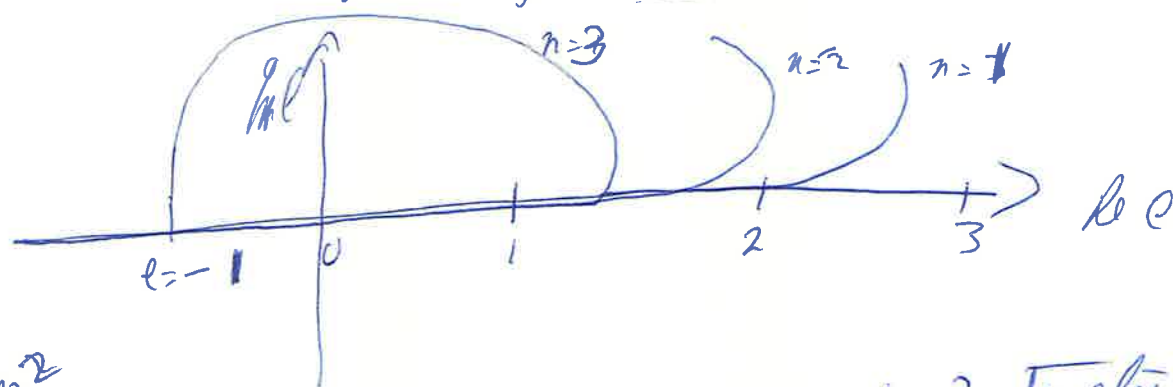
$-\int_{C'}$ is the so-called background integral

n^{th} Regge-pole is at $l = l_n(s)$ 3

plot of complex no $l_n(s)$, parametrized by s
is called a Regge Trajectory

Alternative definition is $\text{Re } l_n(s)$ against $\text{Re } s$

Example of Regge trajectory for Yukawa potential



$s < 4m^2$

$s < 0$, $l_n(s)$ is a real axis, $s > 4m^2$ trajectory
 value of s (L^2) for which $l_n(s) = 0, 1, 2, 3$
 give the bound states. As strength of

potential increases trajectory moves to right
hyper, hyper ples. generate $l=0$ bound states,
 $l=1$ also start to appear

at $s = -\infty$ to $+\infty$ trajectory goes to $l = -1$

Returning to (1) as charge relation and write

4

$$a(l, s) \approx \frac{\beta_n(s)}{l - d_n(s)} \quad \text{near } n^{\text{th}} \text{ pole}$$

Then β_n is residue.

Regge trajectory is then a plot of $d_n(s)$ projected by s and a plot of $\text{Re } d_n$ against s (assumed real)

Then (1) can be written as

$$f(s, \cos \theta) = -\frac{1}{2i} \int_{C'} dl \frac{(2l+1)}{\sin \pi l} a(l, s) P_l(-\cos \theta) \\ = -\pi \sum_n \frac{(2d_n(s)+1) \beta_n(s)}{\sin \pi d_n(s)} P_{d_n(s)}(-\cos \theta)$$

$\cos \theta$ can now become complex, i.e. we can let t be complex — leads to justification for the Mandelstam double dispersion relation, which was original reason for Regge's work

* We are only very rough

$$f_n(\mu) = \mu + \frac{1}{n}$$

10. $f_n(x) \rightarrow x$ for large x

even if n is center

ie. $|f_n(x)| \rightarrow x$ for large x
 have assumed n is large



Now as $z = \text{cer } 0 \rightarrow \infty$. → does this result? 5

$$\rho(z) \sim |z|^e, \text{ at } \text{for complex } l$$

$$\text{Re } l > -\frac{1}{2}, \quad |\rho(-z)| \sim |z|^{\text{Re } l}, \text{ as } z \rightarrow \infty$$

$$\text{Asymptotic integral} \propto |z|^{-1/2} \text{ as } z \rightarrow \infty$$

$$(\text{at } t \rightarrow \infty) \quad \text{Remember } \cos z^2 = 1 + \frac{z^2}{s-4m^2}$$

Let α_1 be leading Regge pole is on with
largest value of $\text{Re } \alpha(s)$,

then α_1 determines behavior of $f(s, \text{cer } 0)$

$$\text{as } \text{cer } 0 \rightarrow \infty \quad \text{and } f(s, t) \text{ as } t \rightarrow \infty.$$

viz, for one pole:

$$f(s, t) \sim -\frac{\pi(2\alpha_1+1)\beta_1}{\sin \pi\alpha_1} (-z)^{\alpha_1}$$

$$\sim c_1(s) t^{\alpha_1(s)}$$

$$\text{where } c_1 = -\frac{\pi(2\alpha_1+1)\beta_1(s)}{\sin \pi\alpha_1} \left(-\frac{z}{s-4m^2}\right)^{\alpha_1}$$

In the t -channel, this means equivalent 6
 as corresponding to $S \rightarrow 0$, $S \rightarrow t$
 $t \rightarrow S$

2. every n th for n th order

$$f(s, t) \sim c_1(t) s^{d_1(t)} \quad \text{--- (1)}$$

for large S

where d_1 is body part in the t -channel.

for n -poles, a power

$$f(s, t) \sim \sum_n \left(\frac{s}{s_0}\right)^{d_n(t)} + c_n(t)$$

where $d_n(t)$ is location of n th pole.

which determine bound states

resonances in the t -channel (where $t \rightarrow 0$)

From (1) by every behavior cons. velocity
 from $R \rightarrow d_1$

From (1) above we have

7

$$f(s, t) \sim c_1(t) s^{d_1(t)} \quad \text{large } s$$

Assume Pomeron pole at d_P demands
and that $d_P(0) = 1$

then $f(s, t) \sim c_1(t) s^{d_P(t)}$

BC cross-section is given by

$$\sigma_{\text{total}} \propto \frac{1}{q} \int_{\text{Im}} f(s, 0)$$

$$s \propto q^2 \quad \text{but } \text{even.}$$

and $\frac{d\sigma}{dt} \propto \left| \frac{f(s, t)}{q^2} \right|^2 = \frac{1}{q^2} |f(s, t)|^2 = \frac{1}{s} |f|^2$

$$\therefore \frac{d\sigma}{dt} \propto \frac{1}{s} |c_1(t)|^2 s^{2 \operatorname{Re} d_P(t)} \approx \frac{1}{s} |c_1(0)|^2 s^{2 \operatorname{Re} d_P(t)}$$

$$\text{or } \left(\frac{d\sigma}{dt} \right)_{t=0} = \frac{1}{s} |c_1(0)|^2 s^{2 \operatorname{Re} d_P(0)}$$

$$\therefore \frac{1}{s} |c_1(0)|^2 = \left(\frac{d\sigma}{dt} \right)_{t=0} e^{-2 \operatorname{Re} d_P(0)}$$

$$\text{and hence } \frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt} \right)_{t=0} s^{2 \operatorname{Re}(d_P(t) - d_P(0))}$$

$$= \left(\frac{d\sigma}{dt} \right)_{t=0} s^{2(d_P(t) - 1) \ln s}$$

s is measured, more general in terms of a parameter s_0

So that we can write -

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt} \right)_{t=0} e^{2(d_p(t)-1) \ln(s/s_0)}$$

for small t $d_p(t) = 1 + \epsilon_p t$

$$\epsilon_p = d_p'(t)|_{t=0}$$

as $\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt} \right)_{t=0} e^{2\epsilon_p t \ln(s/s_0)}$

which is of exponential form for small t

ϵ_p is positive

$$\left(\frac{d\sigma}{dt} \right)_{\text{expt}} = \left(\frac{d\sigma}{dt} \right)_{t=0} e^{2\epsilon_p t \ln(s/s_0)}$$

except that instead of increasing with s logarithmically, (remember t is -ve) and $d_p(t)$ is the real, by analogy with potential scattering).

Here we have a shrinkage of diffraction peak as energy increases, not desired - perhaps present experimental errors not high enough for simple pole approximation to be valid?

$d\rho(0) = 1$ over that $\sigma_{total} \rightarrow$ (constant) σ

$$= \frac{1}{q} \ln e(t)$$

$$= \frac{1}{q} \ln \zeta(0) \cdot \times S$$

$c_1 \propto \frac{1}{q}$ here. Total $\propto c_1(0)$,

and is independent of S .

But fermion is not a true fermion
since it has even signature, which is
not available to $l=1$, is relativistic
theory.

Gribov - Fröman Relativistic Treatment

showed by considering partial wave amplitudes
derived from Nambu-Goto representation (or here)
that $a_e(s)$ does not satisfy conditions for
Lorentz invariance — instead defines two
amplitudes $a^+(l,s)$ and $a^-(l,s)$ such that

$$a^+(l,s) = a_e(s) \quad l=0, 2, 4, \dots$$

$$a^-(l,s) = a_e(s) \quad l=1, 3, 5, \dots$$

then a^+ and a^- can be used for the
unique identification as the states conditions
for offlying Cardan's theorem.

a pole of $a^+(l, s)$ for $l = \text{even value} > 5L4m^2$
corresponds to a bound state, but a
pole of $a^+(l, s)$ for l odd does not,
therefore there is not a bound state
(or free particle)

Bound states poles, resonances

From Regge form for $f(s, t)$

near integral l for pole with $8L4m^2$
we get a pole in f which corresponds
to a bound state

which for $s > 4m^2$ if $d_n(s) = l + \text{Im} d_n(s)$
where $\text{Im} d_n(s)$ is small and

$l = \text{Re} d_n(s)$ we obtain a resonance form
for $d_n(s)$ as discussed in Group. p 485

Concave utilities

% $F(x) = 0$ has roots at x_1, x_2 .
 double root at $x = x_2$ means F has
 form (for polynomial) $F(x) = (x - x_2)^2 \phi(x)$

$$\left. \begin{aligned} F(x_2) &= 0 \\ F'(x_2) &= 0 \end{aligned} \right\}$$

also follows from

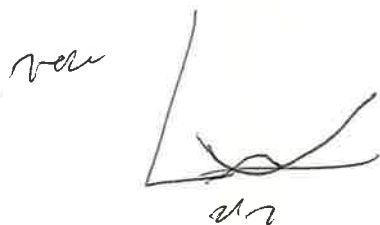
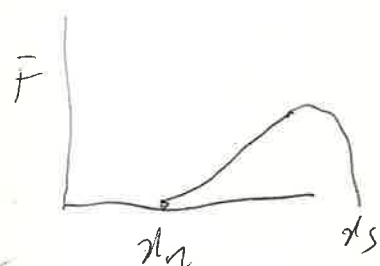
$$F(x_3) \approx F(x_2) + \frac{\partial F}{\partial x_2} (x_3 - x_2)$$

for x_3 near to x_2 but not coincident.

then $0 \approx 0 + \frac{\partial F}{\partial x_2} (x_3 - x_2)$

ie true only if $\frac{\partial F}{\partial x_2} = 0$.

In general.



or no reason why it is a target to
 all come at a double zero

14. Concave zero is
 at condition for F to be a minimum zero.
 but to be minimum

Questions for Carlitz

(1)

① In Goe. invariant to say we must use
 $\frac{\partial H_{\mu}^{(+)}(\Phi)}{\partial x} = 0$ as subsidiary condition

② Definition of S-matrix

Two operators are defined $S \approx V$ such that

$$(\phi_i; S \phi_j) = (\psi_i^{in}, V, \psi_j^{in}) = S_{\alpha\beta} \text{ where } \phi \text{ are eigenstates of } H_0, \psi_i \text{ are eigenstates of } H$$

Then S-matrix element is $S_{\alpha\beta} = \langle \psi_i^{out} | \psi_j^{in} \rangle = \langle \psi_i^{out} | \psi_j^{in} \rangle$

where $S = U(\infty, -\infty)$, but $V \neq S$

& V is used in Y-ay-Yeldman result

$$\phi^{out} = V^{-1} \phi^{in} V$$

But, according to Toul, which

V & S are identical

where is the discrepancy?

a.) Schwinger uses interaction rep. at $t=0$

b.) Toul uses interaction rep. at $t=-\infty$,

and assume identity of $\phi^{interaction}$ & ϕ^{free} in Y-F. formalism.

(3) Jensen L.S.2. solution for $H' = U + V$ along with

(4) In weakly spreading F in $[F, \phi] = \text{It. ex. } \phi$. F is general of independent events

Information is $\int \Pi(x) F^{(c)}(x) dx$. where $F^{(c)}$ are the reference measures. $F^{(c)}$ only the same cumulative density as the $F^{(c)}$ and provide a reference of same.

$$J^{(c)} = \int \Pi(x) F^{(c)}(x) dx$$

(5) In Ellett's current hypothesis (cf. better than function)

and the current from all 8 nodes later are we obtaining full - given that the - of weak spreading - changing coding is between p. 1.

Answer to Q. 2

By definition $V = R^{(+)} R^{(-)-1} = U(0, -\infty) U(\infty, 0)$ (1)

$R^{(+)} = U(0, -\infty)$
 $R^{(-)} = U(0, \infty)$

and $S = U(\infty, -\infty)$

But by J & R. p. 118.

$U(\tau_0, \tau) U(\tau, \tau_0)$ is independent of τ_0 .

Take $\tau = -\infty$
 $\tau_0 = +\infty$

then $U(\tau_0, -\infty) U(\infty, \tau_0)$ is independent of τ_0

$= U(0, -\infty)$ with $\tau_0 = -\infty$

$= U(0, -\infty) U(\infty, 0)$ with $\tau_0 = 0$

$= V$

Hence $S = V$ as

stated in J & R. or directly
 is confirmed.

Note (1) often seen: $\langle \psi_m | V | \psi_m \rangle$
 and we $\langle \psi_m | = U(0, \infty) \langle \phi_m | = \langle \phi_m | S | \phi_m \rangle$
 to give $\langle \psi_m | V | \psi_m \rangle = \langle \phi_m | U(-\infty, 0) V U(0, -\infty) | \phi_m \rangle$
 when $S = U(\infty, -\infty) = U(\infty, 0) U(0, -\infty) = U(-\infty, 0) V U(0, -\infty)$
 where next follows.

⑥ in curved motion do we see how
 p_1, p_2 to p_3, p_4 as straight momenta
 or less curved momenta at p_1, p_2 or
 in our test?

curved in : u-dense

$$p_3 \rightarrow -p_3 \quad p_1 \rightarrow -p_1$$

so $u \rightarrow (p+u)^2 =$

c.f. a
 any
 the curved
 the curved

t-dense

$$p_4 \rightarrow -p_4 \quad p_1 \rightarrow -p_1$$

$t \rightarrow (p+h)^2 =$

c.f. in every
 in the curved column

the method is better the curved one.